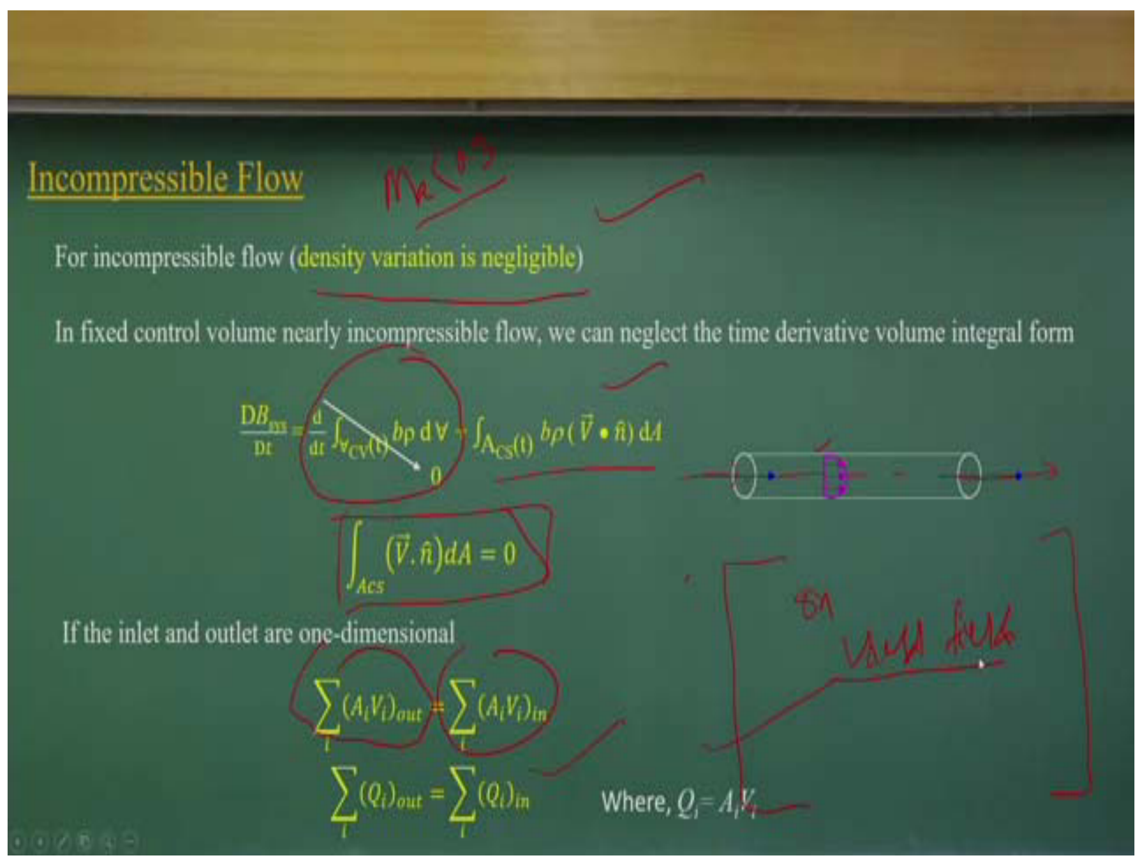

Now, let me come in to the, if the flow is incompressible. Again, we have a lot of simplifications. Again, I can repeat it. The flow systems when you have mac number less than 0.3, okay, whether it is gas, whether it is a liquid or any flow system, if you think that the within the flow system the flow becomes less than the mac number less than 0.3, then there will be density variation, but that variation of density is much much negligible comparing to other components.

So, we can assume the flow is incompressible nature. Again, I am going to summarise that. When you have any flow systems, mac number is less than 0.3, so we can use flow as incomprehensible flow, density does not vary significantly. So, density becomes constant, as density becomes constant, as you know it, it is very simplified problem what we are going to solve.

So, density varies negligible, as the density variation is not significant and beta equal to 1, so only this equ ation is left for us. Simple thing. This is very simple equation.

$$\frac{\mathrm{D}B_{sys}}{\mathrm{D}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\forall_{CV(t)}} b\rho \, \mathrm{d}\forall + \int_{A_{cs(t)}} b\rho\,(\vec{V} \bullet \hat{n})\, \mathrm{d}A$$

$$\underbrace{\quad}_{\rightarrow\, 0}$$

$$\int_{Acs} (\vec{V}.\hat{n})dA = 0$$

Now, there, the scalar product of V and n and d A, okay, and density can come out. So, instead of the mass flux we are now talking about volumetric flux. That means, if you multiply the velocity into area, then what you get is unitm^3/s, volume per unit height, volumetric flux, okay? So, please do not confuse, this is a different equation.

If the inlet and outlet are one-dimensional

$$\sum_i (A_i V_i)_{out} = \sum_i (A_i V_i)_{in}$$

$$\sum_i (Q_i)_{out} = \sum_i (Q_i)_{in}$$

Where, $Q_i = A_i V_i$

Only that we are not showing the density multiplication. If you multiply with a Q, the V into A is Q is discharge. So, Q = V $\times A$, is the discharge. So, most of the conservation of mass you write it, since density is a constant, you make it come out from that equation.

So, it looks like volumetric level we are comparing but all are mass conservation equations. We talked about mass flux is coming in or going out from the control volume or mass flux is changing within the control volume. That is the concept to that. But as it is simplified, in case the flow is incomprehensible, density is a constant, that density component comes out from Reynolds transport theorem which helps us look like we are looking at equating the volumetric thing, but it is not that.

Please remember we are still doing the mass conservation equation. The volumetric form has come in because you have taken out the density. That is what if you look it A_1 V you have these things. That means, if I have a pipe flow like this, you can anticipate it. As we discussed earlier, the velocity will be 0 near the wall, velocity will be maximum at the center and so there will be velocity distribution, there will be velocity distribution from this side.

The flow coming and going out. If this is simple in and out system, you can know this velocity distribution area, find out the discharge from inflow and outflow, equate it, then you can solve the problem. So, what I am looking at is again summarised here. To solve this mass conservation equation I should have knowledge on velocity field. I should know how the velocity varies or I should know whether the velocity is a constant or the velocity varies. If I know the velocity variations on this control surface, then I can solve the problem.

So, basically, when you apply the mass conservation equation your knowledge of velocity variation is important to you, how you are simplifying the velocity field on the control surface. That way we solve the problem.

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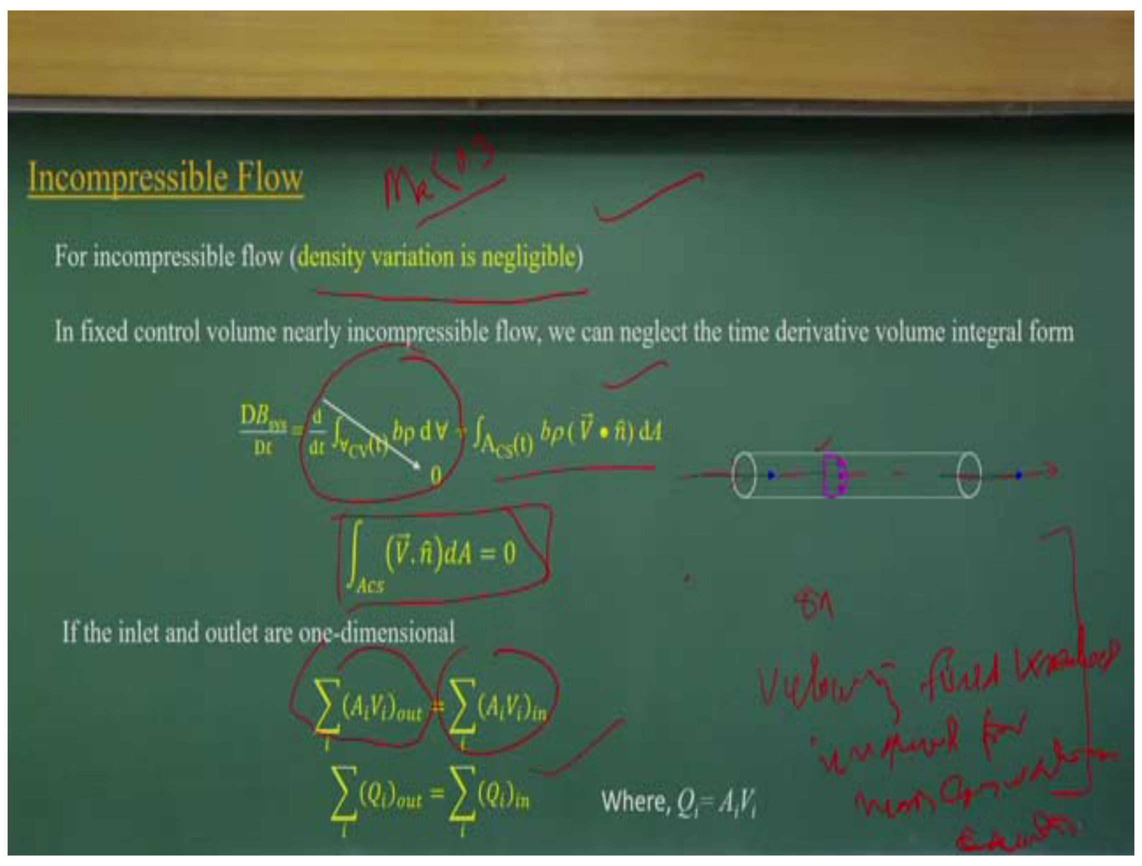

So, the velocity field knowledge is required for mass conservation equations.

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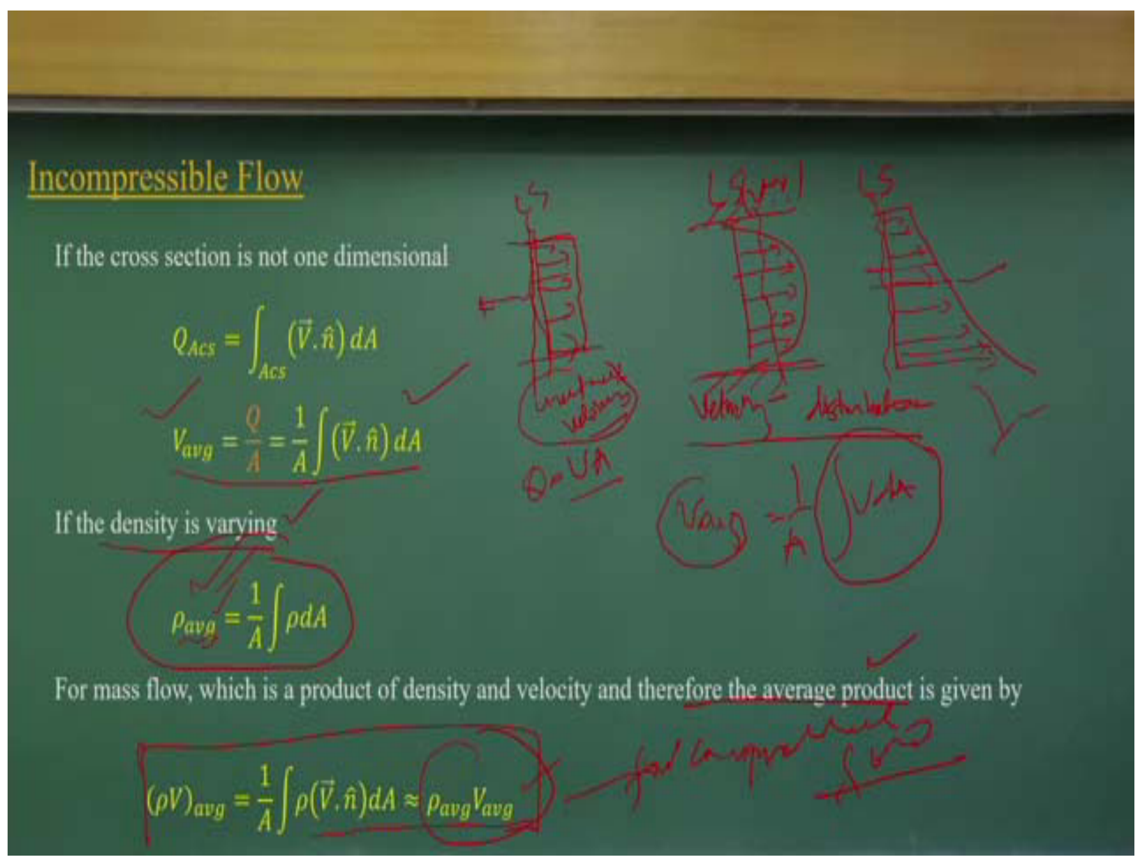

So, that way, whenever you take fluid mechanics problems, first you think what could be approximate velocity field, what could be the velocity direction. If I assume the flow is one dimensional, is it enough for me that one dimensional flow is okay for us or not, for that problem or not, or you need to have two-dimensional velocity fields, or what could be the direction, whether it is a direction with respect to the control surface normal vectors.

All the knowledge you should have when you are applying any real life fluid problems, but as I said earlier, this academic problem is simplified and they do not tell the velocity component, whether the velocity what they talk about that is perpendicular to the control surface, that way it is considered. So, that is the easier way it is there for knowledge about the velocity fields, that is what is necessary.

And where you have velocity fields are not known, I do not think you can apply the conservation of mass equation properly. So, let us come into the incomprehensible flow. Let us look at these figures, you can assume it uniform velocity distribution, okay? That means velocity does not vary with respect to the position. It is a constant velocity. If it is a constant velocity, V into area will give me the Q value.

$$Q_{Acs} = \int_{Acs} (\vec{V}.\hat{n})\, dA$$

It is very easy, V into area will give its value. Only I need to have the control surface where the normal vector should have either velocity vector directions and this normal vector, either 0° or 180° to find out whether it will be Q positive or Q negative. So, when you have uniform velocity, the problem is quite simplified. But the case is you do not have uniform velocity as you know it when you have a pipe flow.

You cannot have uniform velocity. You do not expect that you will have uniform velocity for that. So, you will have 0 velocity near the boundary of the wall of the pipe and you will have velocity like this. So, in that case, many of the problems will give you average velocity. The average velocity what we get,

$$V_{avg} = \frac{Q}{A} = \frac{1}{A} \int (\vec{V}.\hat{n})\, dA$$

So, it represents average velocity. It considered the velocity distribution to compute the average velocity. So, somewhere the average velocity will come like this okay. So, some of the case, the problems give the average velocity. That means it considers the velocity distribution, after that it has given the average velocity. By doing surface integrals average velocity is given.

Once you know the average velocity you multiply with the area you will get the volumetric flux or the discharge. But in some of the cases, if you know the velocity distributions, how it varies. You do surface integrals of that and find out what is average velocity. Multiply with the density and the area will get the mass flux. We will get mass flux if you have the product of density, velocity, and the area. Velocity will be the average velocity.

In some cases like this, you may have the condition where the velocity variations may be very complicated. Then, you need to do the integration to compute the average velocity for that. So, we need to do surface integrals to compute it, how the velocity is varying it and this is my control surface. So, that way we can quantify the V average which is by integrating velocity.

Similar way, consider if the flow is comprehension, you assume the density is varying it, you can use average density concept, okay? Again, you can integrate density with respect to area, then you compute the average things and you can multiply that. Please remember in this case it cannot be if a density and the velocity, the multiple functions will not be separate functions, okay?

They will be depending on each other. Then, this simplification cannot be done. But assuming the density and the velocity does not depend on each other, they are independent, then you may follow this concept to do this averaging to find out for compressible flow, okay? For comprehensible you can do this average product. But the assumption is that the density does not have dependency with the velocity vector.
If the density is varying

$$\rho_{avg} = \frac{1}{A}\int \rho dA$$

If it is that, then you need to do the product of the velocity and the density and do the surface integral to solve the problem.
For mass flow, which is a product of density and velocity and therefore the average product is given by

$$(\rho V)_{avg} = \frac{1}{A}\int \rho(\vec{V}.\hat{n})dA \approx \rho_{avg}V_{avg}$$

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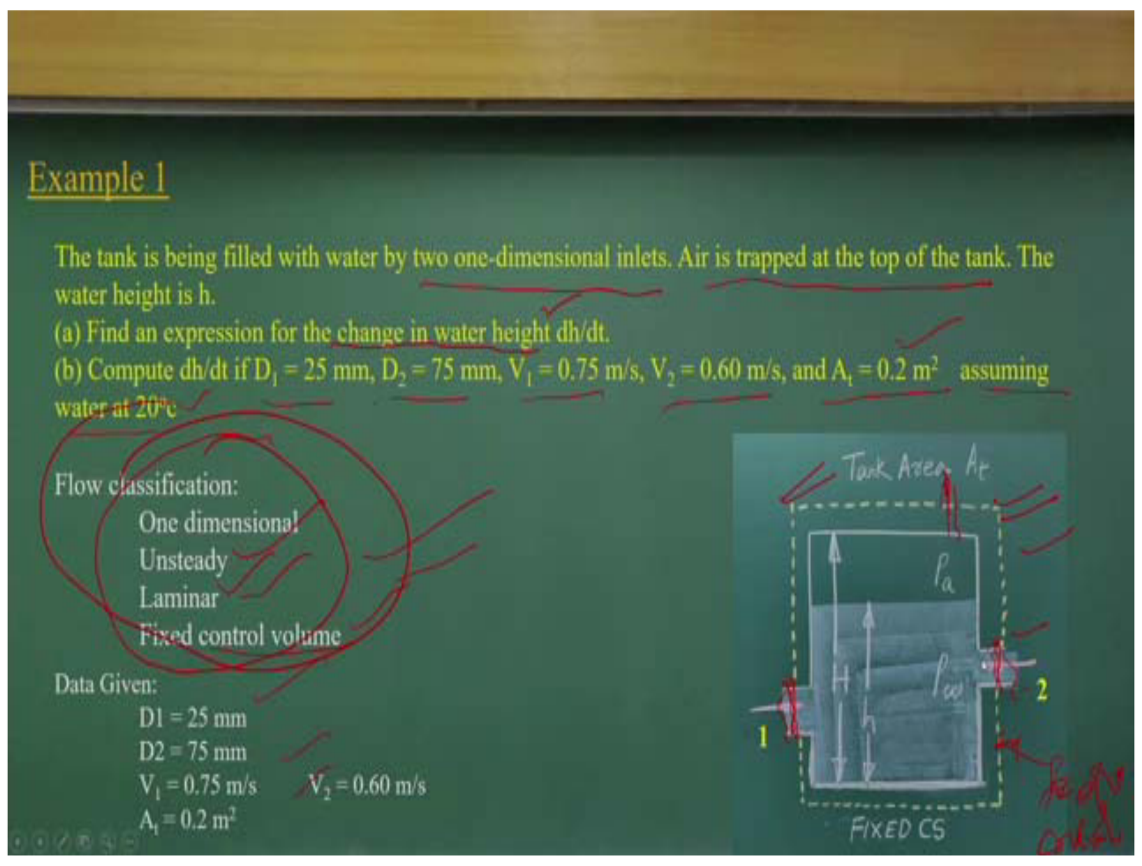

Now, let us come to the very interesting problem which is there in text book of F.M. White book. What is there, there is a tank. If you look at this figure, there is a tank, two inflows are there, the tank is being filled with waters. Within the tank there is air and there is water, liquid and gas form of water. The problem is not given here but I can put it there should be air valve here, okay?

The tank is being filled with water by two one-dimensional inlets. Air is trapped at the top of the tank. The water height is h.

(a) Find an expression for the change in water height dh/dt.

(b) Compute dh/dt if D_1 = 25 mm, D_2 = 75 mm, V_1 = 0.75 m/s, V_2 = 0.60 m/s, and A_t = 0.2 m^2 assuming water at 20^{o}c

As the liquid expands it, there should be air valve for air to come out from that. Otherwise, this will be very complicated problem. As you put more and more liquid, but the air cannot be compressed, that type of problem we are solving. Having said that there is air valve, as the liquid increases the space and the gases are not able to fit there, it will come out, okay? That is very simplification we have to do it for this problem.

Flow classification:

- One dimensional
- Unsteady
- Laminar
- Fixed control volume

So, this is one-dimensional inlet. Air is trapped at the top of the tank with air valve, what I have included here, and this water height is h, find the expression change in the water level with respect to time. How water level is changed?

As you know it, the density also changes with temperature and the pressure, but mostly for the liquid like water it is temperature dependent. So, that is what we are doing here. So, given data is here. But again I need to tell you when you solve the problem, first you do the flow classification. The problem here is one-dimensional in nature. Unsteady because we are finding out the storage varies with respect to time.

Flow can be laminar or turbulent, we do not know it, okay, and we have a fixed control volume. This is what we consider is fixed control volume, okay? First, whenever you solve the problem, you classify the problem. Once you classify problems it gives indirectly that these are the assumptions that are valid for this problem that we are solving. So, classify the problem. Give the data given.

Data Given:

$D1 = 25$ mm

$D2 = 75$ mm

$V_1 = 0.75$ m/s $V_2 = 0.60$ m/s

$A_t = 0.2\ m^2$

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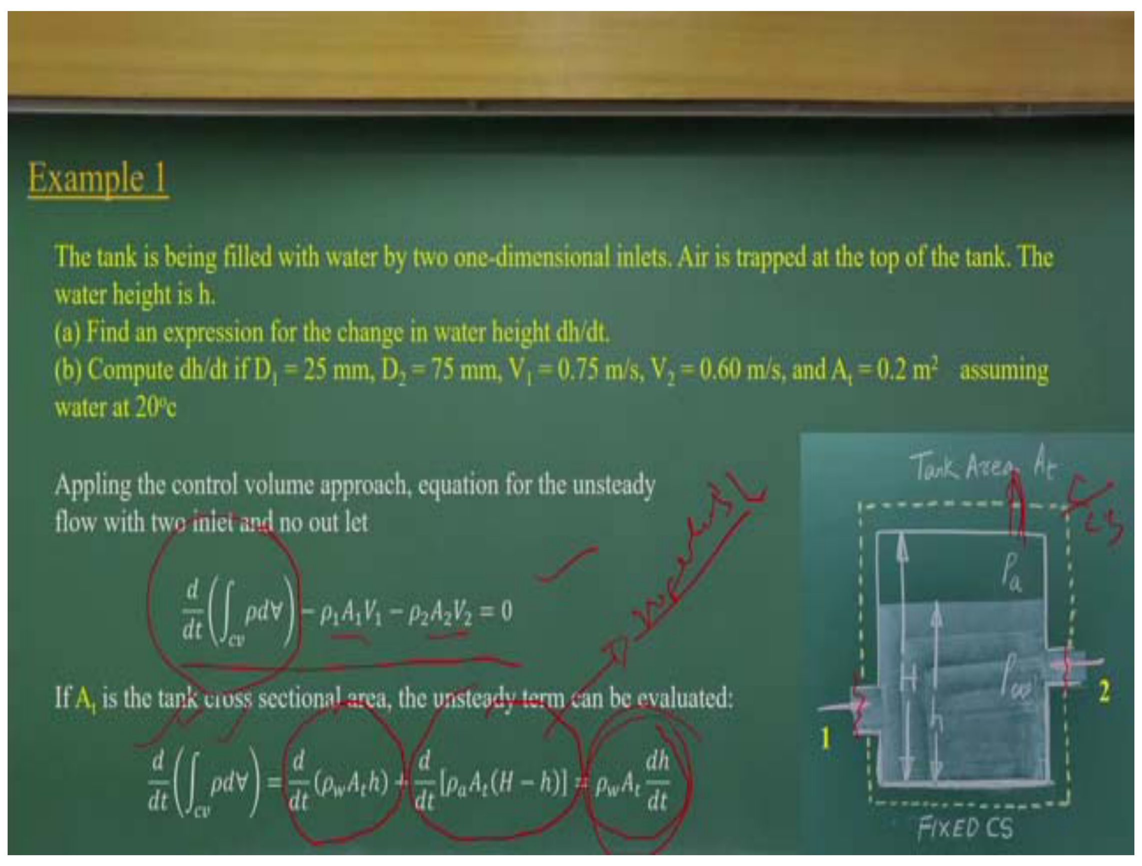

If it is that, apply the Reynolds transport theorems, okay? You apply the Reynolds transport theorems. You have the inflow and the outflow. There is no outflow in this case. In both the case you have inflow, which is negative here. So, rho 1 A1 A2 and rho 2 A2 V2 and what is changing the storage of the water inside this control volume. This is the control volume and this is control surface. The flow is coming in, okay?

Appling the control volume approach, equation for the unsteady flow with two inlet and no out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) - \rho_1 A_1 V_1 - \rho_2 A_2 V_2 = 0$$

What is happening is there is change of the mass of water here, change of the air here. This part we are neglecting it because we are not putting this air valve here. We are neglecting that part, how the air part is changing, otherwise we will go for incompressible and all. Do not go for that, that is complicating this problem. So, we are just talking about how this part is changing it. So, you can simplify it.

If A_t is the tank cross sectional area, the unsteady term can be evaluated

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) = \frac{d}{dt}(\rho_w A_t h) + \frac{d}{dt}[\rho_a A_t (H-h)] = \rho_w A_t \frac{dh}{dt}$$

The density does not change; it is incompressible flow. We know the area of the tank. So, dh by dt, that is how this integral and surface integral will come to this one. Now, we will equate that.

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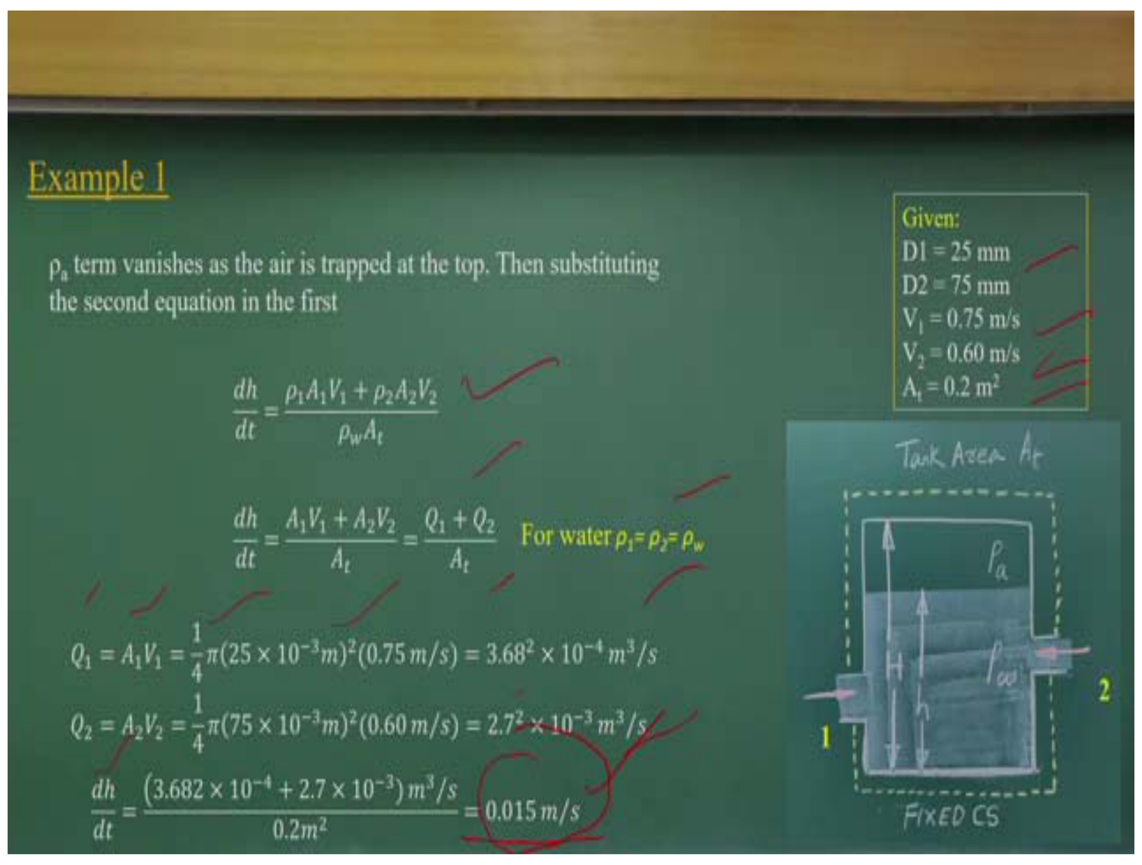

ρ_a term vanishes as the air is trapped at the top. Then substituting the second equation in the first

$$\frac{dh}{dt} = \frac{\rho_1 A_1 V_1 + \rho_2 A_2 V_2}{\rho_w A_t}$$

Again, I am going to repeat it. First, you classify the problem. Assume the appropriate control volume and the control surface and each control surface you identify which is the inflow and outflow. Then, you talk about what is the change of the storage and what is happening.

So, if you can understand properly, applying this Reynolds transport theorem and the simplification and put the numerical value, it does not take much time or much problem to solve any problem. The first is the classification and application of appropriate control volumes. Find out the influx what is coming, mass influx, from where and from which part of the cross section, like this part is inflow, this part is also inflow, and the change in the storage is here. And the change in storage what we can represent.

And if we do that, the problems you can solve it. So, first how to use the classification. Again, I am going to repeat it. Please do the classification accurately. Then, choose appropriate control volume. The control surface should be perpendicular to the velocity vector. That is what you see there. I can have any shape, then I have to do surface integrals. To avoid the surface integrals we have to make control software so that velocity should be perpendicular to that, okay.

With respect to normal vector it should have 0°or 180°. So, in that case, you need not do surface integrals. So, your control surface you chose in such a way that you should have the normal vector of the surface and the velocity they should be collinear. That is the concept to be considered. Then, it is very simple simplification, you just substitute the values;

$$\frac{dh}{dt} = \frac{A_1V_1 + A_2V_2}{A_t} = \frac{Q_1 + Q_2}{A_t}$$

For water $\rho_1 = \rho_2 = \rho_w$

$$Q_1 = A_1V_1 = \frac{1}{4}\pi(25 \times 10^{-3}m)^2(0.75\,m/s) = 3.68^2 \times 10^{-4}\,m^3/s$$

$$Q_2 = A_2V_2 = \frac{1}{4}\pi(75 \times 10^{-3}m)^2(0.60\,m/s) = 2.7^2 \times 10^{-3}\,m^3/s$$

$$\frac{dh}{dt} = \frac{(3.682 \times 10^{-4} + 2.7 \times 10^{-3})\,m^3/s}{0.2m^2} = 0.015\,m/s$$

Finally, you will get the rate of change of the height, that means in this the height will change 0.015 meter per second. The storing within the tank will be there like this, okay, 0.015 meter per second. Let us come to the second example which is very interesting example, showing you the facilities what we have in IIT Guwahati, in the department of civil engineering.

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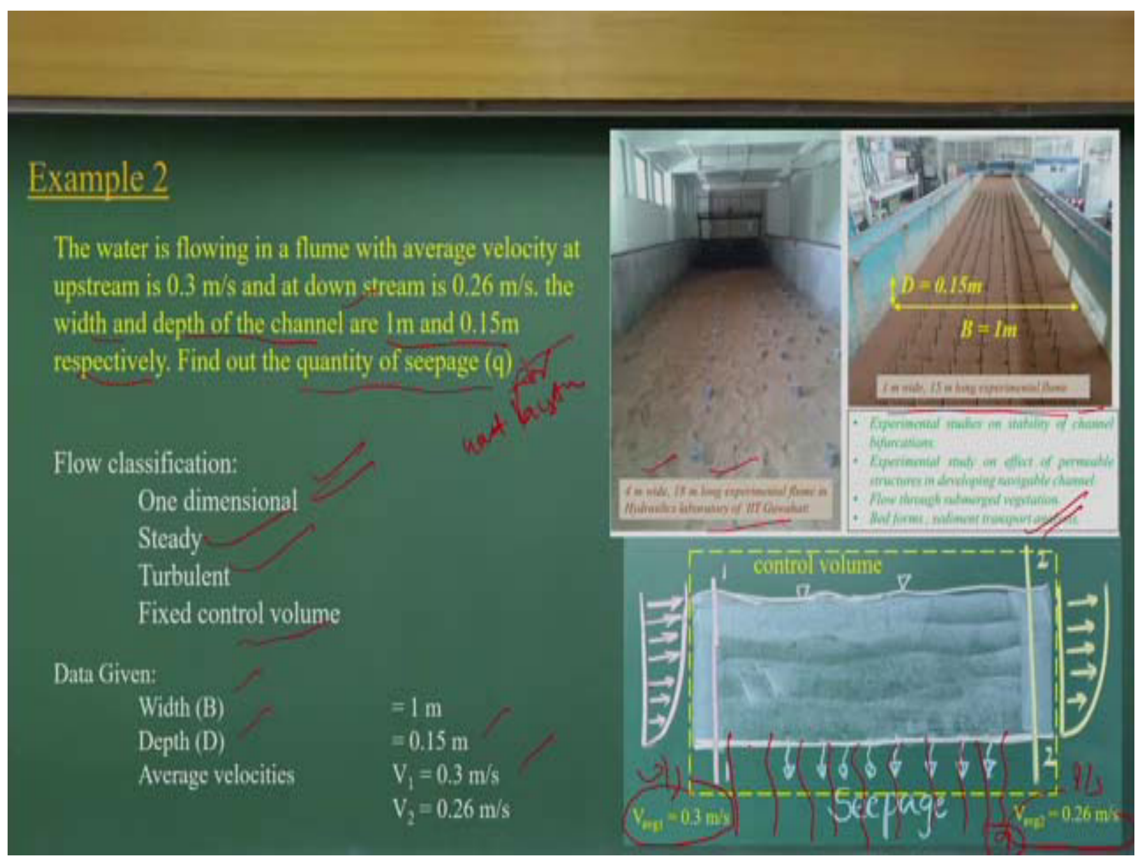

The water is flowing in a flume with a downward seepage average velocity at upstream is 0.3 m/s and at down stream is 0.26 m/s. the width and depth of the channel are 1m and 0.15m respectively. Find out the quantity of seepage (q).

That is experimental flume 4 meter wide and 18 meter long. The photograph you can see. Similar way, we have the experimental facility to do hydraulic studies, having 1 meter wide and 15 meter width. Here we have the seepage arrangement which is unique in this way. So, that seepage arrangement means water from the surface can go to the ground water. That means, from this control volume water can seep downwards.

That is the seepage facility what we have here. Considering that is what the control volume is, we are now going to solve this problem that if I have the velocity measurement at the upstream and the downstream, can I compute what will be the seepage rate per unit length, okay? That means I know the average velocities at the upstream, I know this downstream velocity.

Also I know the width and the depth of the channels respectively, then can I compute it how much of water goes out from this control volume at seepage as a downward movement. Flow classification:

One dimensional

Steady

Turbulent

Fixed control volume

So, this is how we have a control volume. So, this is the upstream direction and this is the downstream direction. This is my control volume. Some of the water seepage out from this control volume.

Data Given:

Width (B) = 1 m

Depth (D) = 0.15 m

Average velocities, $V_1 = 0.3$ m/s

$V_2 = 0.26$ m/s

Mass influx, mass outflux since it is a steady problem, and density is constant. So, at volumetric level I just compare it, how much of water is coming to this control volume, how much of water goes as seepage, as a downstream, outside from this control volume.

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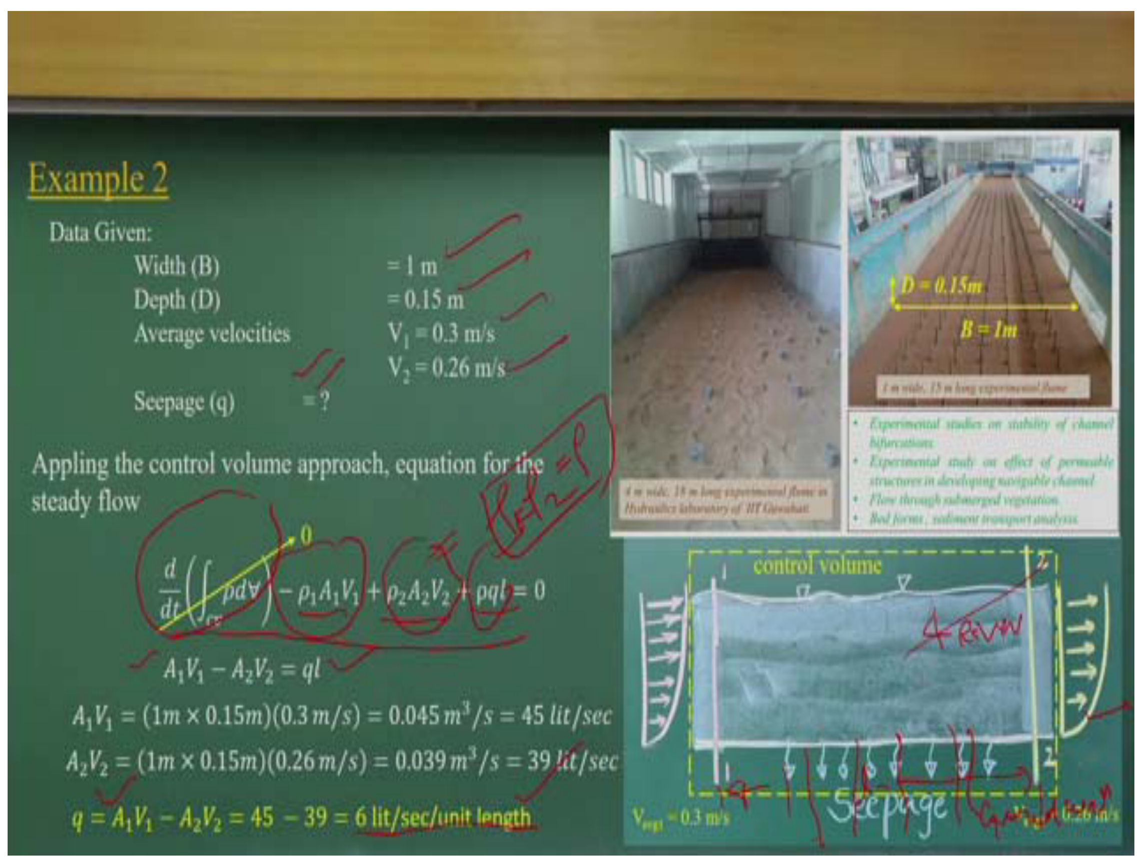

Now, we are getting this width and depth. We have the velocity, then we are looking at the seepage rate. You can see I apply the mass conservation equations. Data Given:

Width (B) = 1 m

Depth (D) = 0.15 m

Average velocities, $V_1 = 0.3$ m/s

$V_2 = 0.26$ m/s

Appling the control volume approach, equation for the steady flow

0

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) - \rho_1 A_1 V_1 + \rho_2 A_2 V_2 + \rho q l = 0$$

This is the outflux going out from this. This is outflow, seepage, downward path. So, if you rearrange it, you get the ql is this as the density is a constant. The water density does not change in this flow system. So, you get what will be the q value liter per second per meter. That is what you compute. Only we have to visualise that. There is the flow, inflow and outflow, the seepage.

$$A_1V_1 - A_2V_2 = ql$$

This is the same way in a river. If this is the river, this is the groundwater zone. There is exchange, river recharges to the groundwater. This same process happens, or reverse also is true, ground water can give flux into the river. That means this q will be positive or negative sign. Otherwise, with the same control volume we can apply mass conservations for a river and groundwater interaction study. That is what you learned in hydrology course.

$$A_1V_1 = (1m \times 0.15m)(0.3\, m/s) = 0.045\, m^3/s = 45\; lit/sec$$

$$A_2V_2 = (1m \times 0.15m)(0.26\, m/s) = 0.039\, m^3/s = 39\; lit/sec$$

$q = A_1V_1 - A_2V_2 = 45 \; - 39 = 6$ lit/sec/unit length

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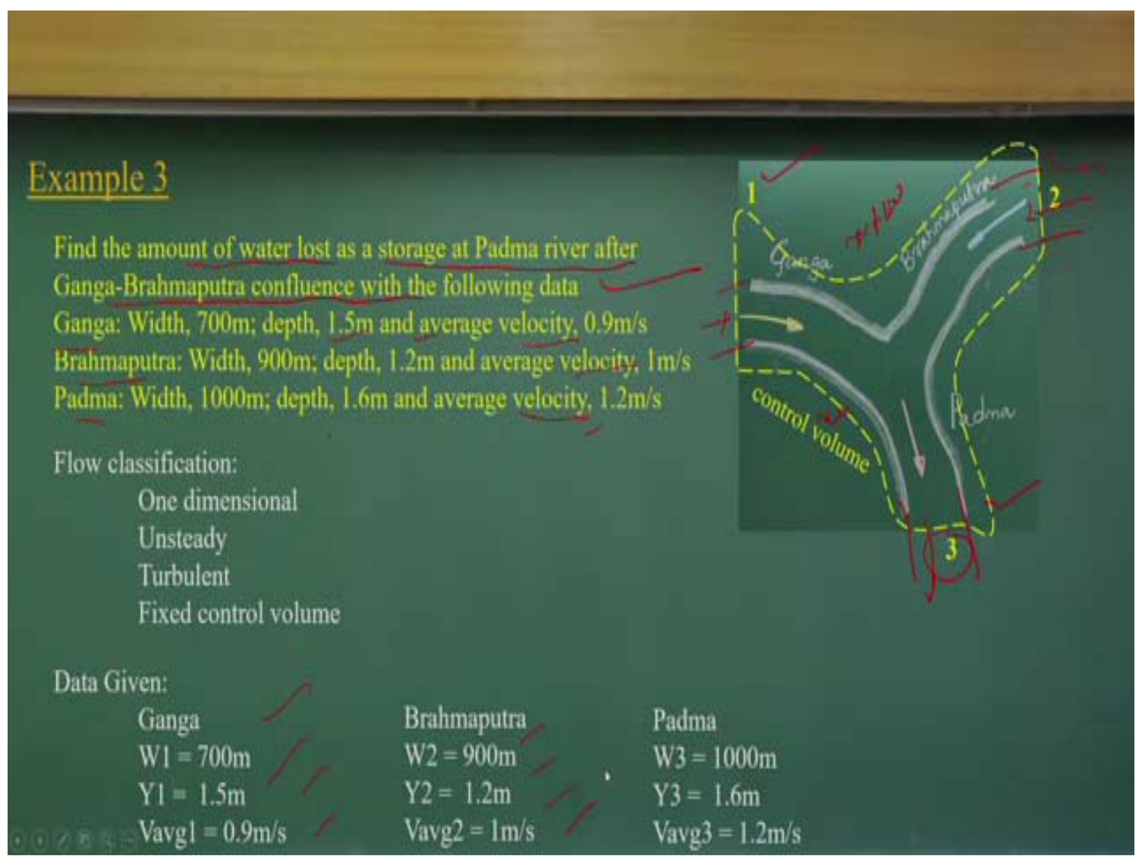

Now, take another example, problem which is with Ganga-Brahmaputra confluence which is not in India, it is in Bangladesh, okay. Let us consider Ganga and Brahmaputra is meeting, okay? We have Padma river systems, and with these rivers we have all the measurements available of these systems. At 1, 2, and 3, when Ganga and Brahmaputra and Padma meets there we have a cross section 3 here.

Find the amount of water lost as a storage at Padma river after Ganga-Brahmaputra confluence with the following data

Ganga: Width, 700m; depth, 1.5m and average velocity, 0.9m/s

Brahmaputra: Width, 900m; depth, 1.2m and average velocity, 1m/s

Padma: Width, 1000m; depth, 1.6m and average velocity, 1.2m/s

So, we have a control volume like this. So, this is a no flow, this is a no flow. The flow will be only this, here and here and here. So, this is inflow, this is inflow, this is outflow. So, the average velocity is given, width is given, depth is given. So, we have to compute the q_1, q_2, and q_3. Based on that we have to find out whether change in storage is there or not. That means we have to find out amount of water lost as storage in Padma river after the Ganga-Brahmaputra confluence with the following data.

Flow classification:

- One dimensional
- Unsteady
- Turbulent
- Fixed control volume

Data Given:

Ganga

- W1 = 700m
- Y1 = 1.5m
- Vavg1 = 0.9m/s

Brahmaputra

- W2 = 900m
- Y2 = 1.2m
- Vavg2 = 1m/s

Padma

- W3 = 1000m
- Y3 = 1.6m
- Vavg3 = 1.2m/s

In Ganga, Brahmaputra, and Padma which is are confluencing, after that we call Padma. How much the flow depth is there, width is there, average velocity is given which is more or less average velocity as we do a lot of river survey in Ganga and Brahmaputra systems in our country.

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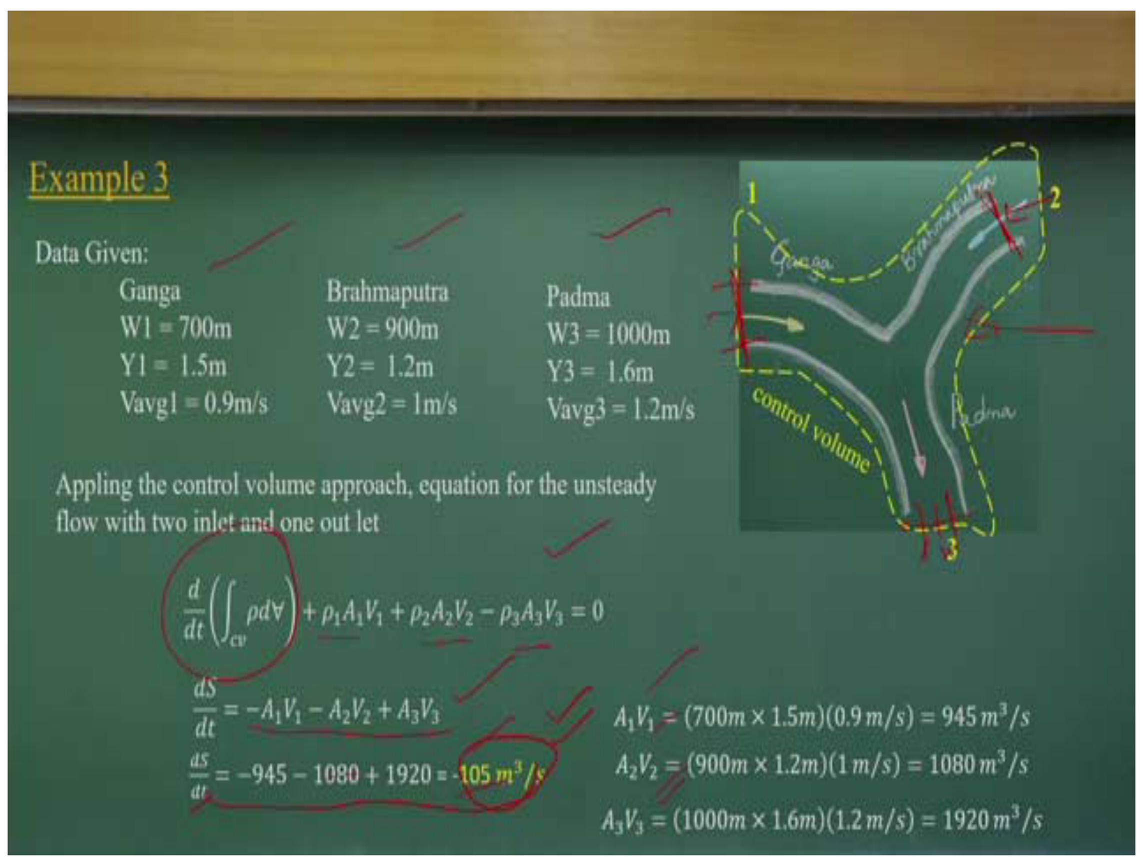
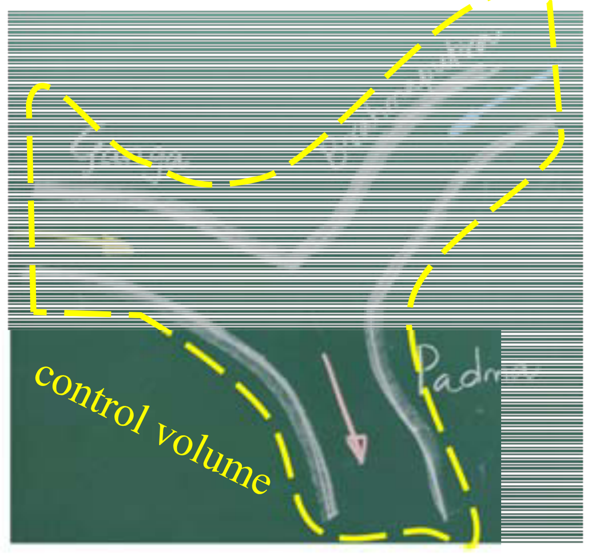


So, if I have width and the depth and velocity like this, so we can apply this continuity equations for this control volume, okay? For Ganga, Brahmaputra, Padma systems I have control volumes and all, I can have positive and the negative, okay?



Appling the control volume approach, equation for the unsteady flow with two inlet and one out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) + \rho_1 A_1 V_1 + \rho_2 A_2 V_2 - \rho_3 A_3 V_3 = 0$$

So, substituting this value with have a negative or a positive of change in the storage. So, simple way we can just multiply it to get the q1, q2, and q3. If you look it is a very complex problem, but with the help of the control volume we just look at mass flux coming in from this surface and going out this surface. That will be change in the mass within this control volume. That is what it says, how much storage of mass is changing within control volume.

$$\frac{dS}{dt} = -A_1V_1 - A_2V_2 + A_3V_3$$

Considering the measurement of the velocity and the flow depth and the area at these three points we can judge how much of water we are losing or gaining in a stretch of river systems. You can understand these type of problems we can solve, okay? Whether river is gaining from other locations or the water is losing from the river. That type of study with a simple type of control volume we can do it.

$$A_1V_1 = (700m \times 1.5m)(0.9\,m/s) = 945\,m^3/s$$
$$A_2V_2 = (900m \times 1.2m)(1\,m/s) = 1080\,m^3/s$$
$$A_3V_3 = (1000m \times 1.6m)(1.2\,m/s) = 1920\,m^3/s$$

$\frac{dS}{dt} = -945 - 1080 + 1920$ = -105 m^3/s

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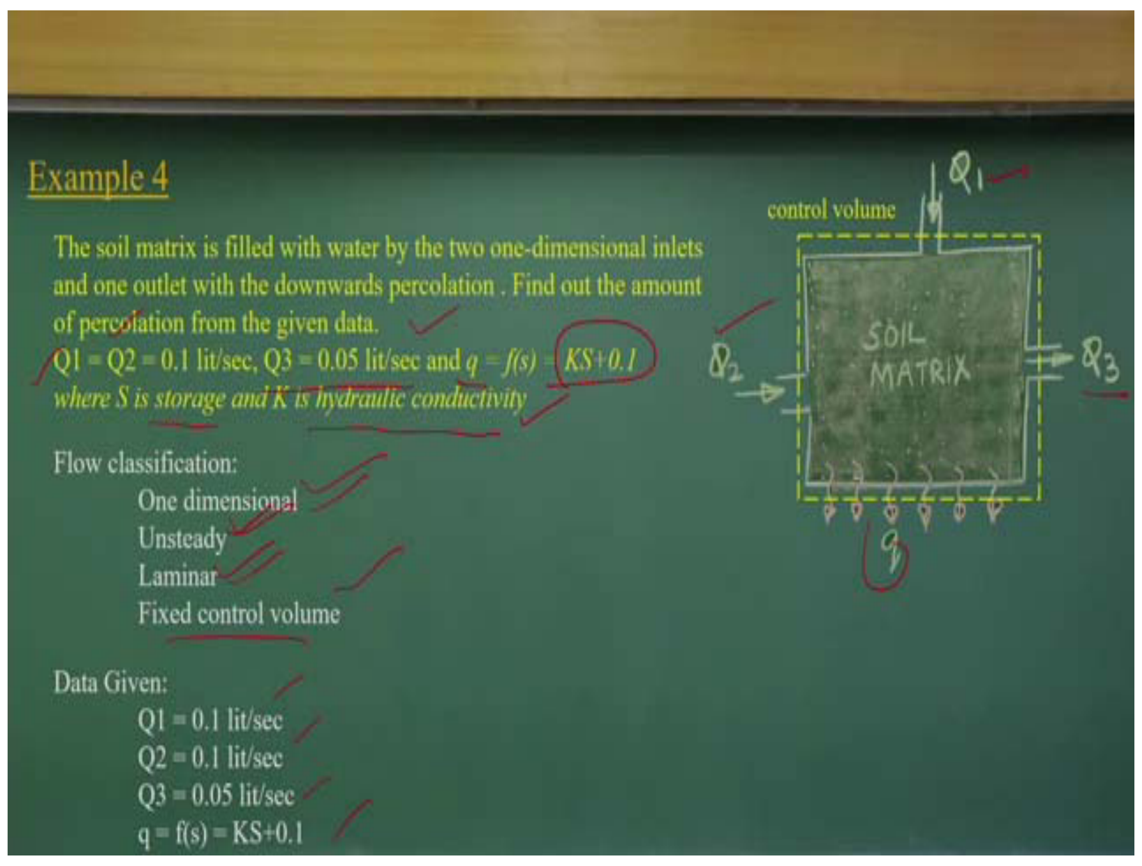

Let us have the last example, okay? It is slightly a bit complicated. There is a soil matrix, let it have chambers. The flow is coming Q1, Q2, and Q3. There are two inflows. The discharge is coming, Q3 is outlet, and there are the seepage flow or download percolations are happening which depends upon the storage within the system. That is why Q is a function of storage.

[The soil matrix is filled with water by the two one-dimensional inlets and one outlet with the downwards percolation . Find out the amount of percolation from the given data.

Q1 = Q2 = 0.1 lit/sec, Q3 = 0.05 lit/sec and *q = f(s) = KS+0.1*

where S is storage and K is hydraulic conductivity]

How much of water in storage, that function with a K, K is linear coefficient that is where which may be considered sometimes as hydraulic conductivity in storage. I have Q1 and Q2, I have the Q3, how of litre per second, because there is very less quantity of water, not comparable with the Ganga and Brahmaputra systems. So, litres per second we are coming to that. So, in that case, flow can confine.

Flow classification:

- One dimensional
- Unsteady
- Laminar
- Fixed control volume

Data Given:

- Q1 = 0.1 lit/sec
- Q2 = 0.1 lit/sec
- Q3 = 0.05 lit/sec
- q = f(s) = KS+0.1

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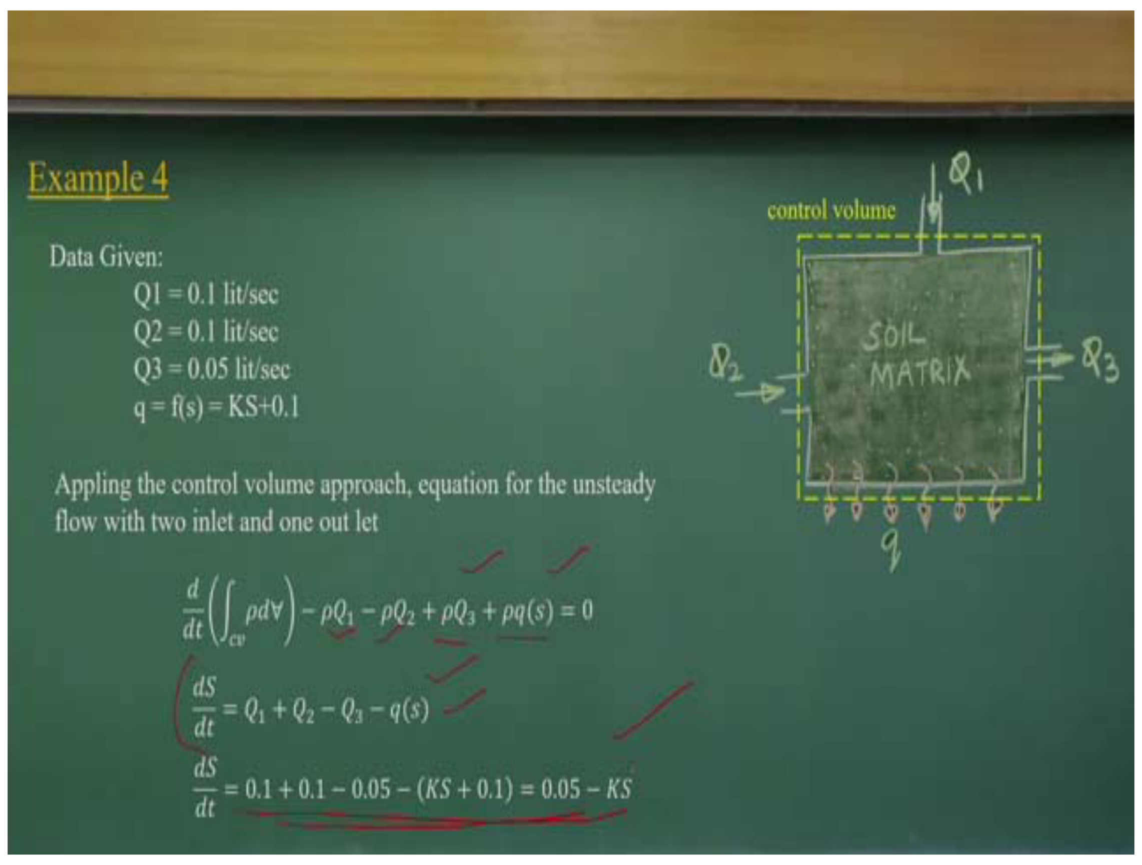

Now, if I substitute this equation, if you look at these inflows are the negative, outflows are positive, and Qx is the function of this. So, Appling the control volume approach, equation for the unsteady flow with two inlet and one out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) - \rho Q_1 - \rho Q_2 + \rho Q_3 + \rho q(s) = 0$$

$$\frac{dS}{dt} = Q_1 + Q_2 - Q_3 - q(s)$$

$$\frac{dS}{dt} = 0.1 + 0.1 - 0.05 - (KS + 0.1) = 0.05 - KS$$

This is in terms of s. K is a constant.

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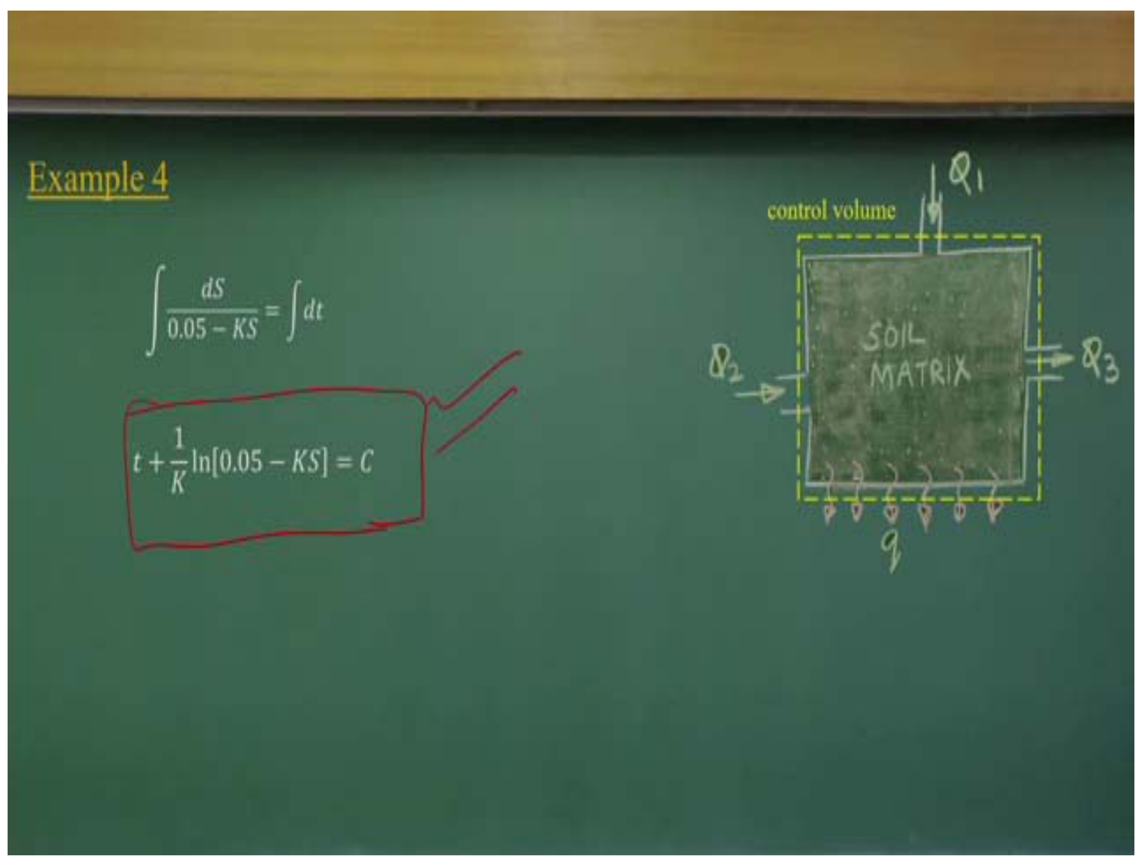

So, we can integrate it to solve these problems.

$$\int \frac{dS}{0.05 - KS} = \int dt$$

$$t + \frac{1}{K}\ln[0.05 - KS] = C$$

So, finally you get this equation. That is how s varies with respect to time. That is what is our problem. So, I have given three examples. One is a simple tank problem. Another is three river confluence point. Third is seepage problem. And fourth is the soil matrix problem. So, that way if you look at any of the Cengel, Cimbala, or F.M. White book, a lot of exercises are there, there are a lot of example problems which are also solved.

So, only this art of applying this control volume concept that you should learn it. May be very complex problems but use of appropriate control volume and the control section knowing the direction of the flow that will help us. By applying the Reynolds transport theorems, we can solve the problems. That is my idea and that what I need to convince.

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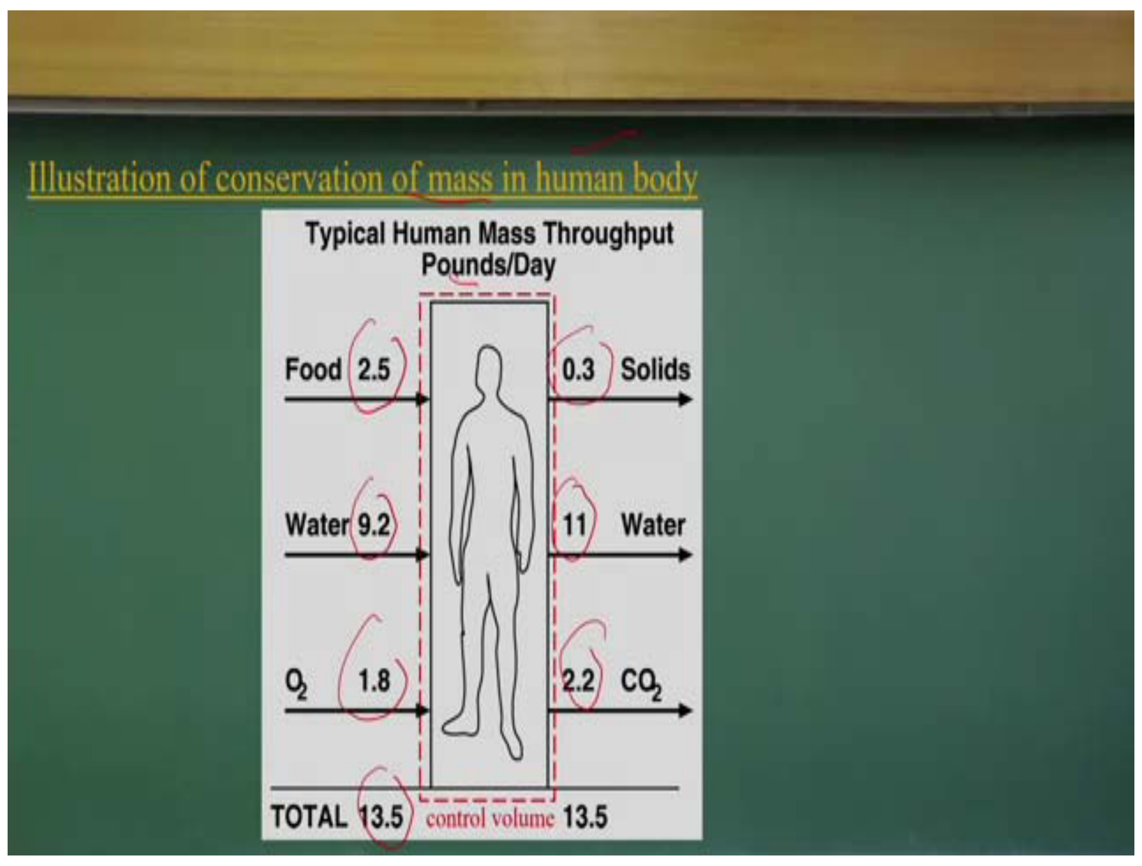

Before that let me give a warning to you or suggestions to you, that whatever human body I or you, all are having mass inflow and outflow systems, that is what it is saying, that the conservation of mass in a human body is pounds per day, 2.5, 9.2 pounds. This much of water and food and oxygen we take, which is 13.5 pounds, okay. And we release the solids, water, and CO_2, some of this 13.5.

Please do not disturb this inflow and outflow. If you disturb the inflow and outflow we do not know what we are doing to our system of body, okay? Either we are deteriorating the body, our health, we do not know it. What I am going to tell is the 9.2 pounds of water please drink it. Similar way, the 2.2 pounds per day of food you eat so that your systems would be perfectly okay for now and total youthful life.

You can enjoy it if you maintain the simple balancing the mass conservation principle is followed by the human body with different water, food, and this, with a slight bit variations. But overall this equation you should follow, with food 2.5, water 9.2, oxygen 1.8 pounds per day. And we release the same amount whatever we get it and release the same amount, only we vary the solids to 0.3, the water to 11, and 2.2.

So, many of the foods we convert to water and we convert the oxygen to carbon dioxide. That we do it and for a healthy life we should follow this equations and we should remember this equation. Whenever you wake up in the early morning we should also maintain this equation, then we will have a healthy life. It is more important to say this.

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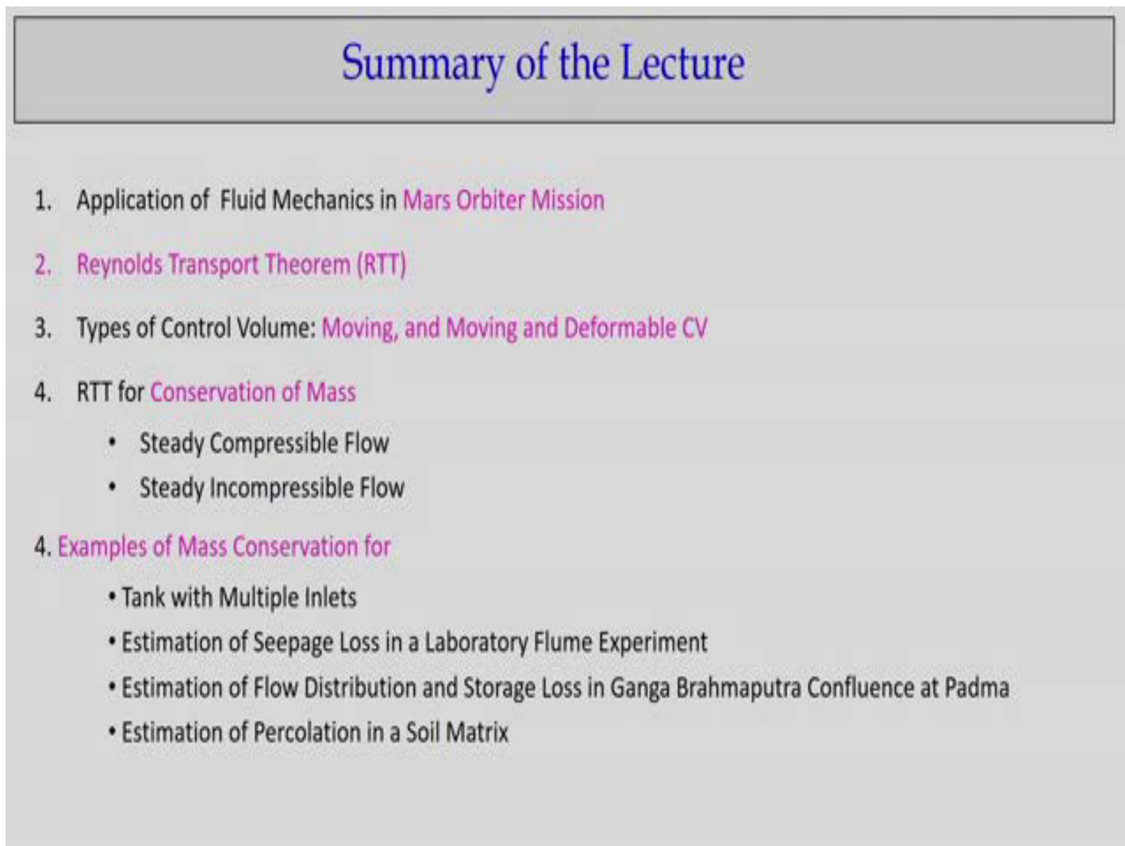

With this let me conclude this very interesting lecture. I do not know whether you enjoyed or not, but let us have a talk about that. We have applied Reynolds transport theorem which looks very difficult, surface integrals or volume integrals, but it can be simplified in many ways and when you simplify this complex equation and apply to real life problems like as I have given examples.

Similar way, real life problems if we can apply it really we can find out what will be change in the storage, what will be the change in mass inflows and outflows, that is a standard problem. So, please try to solve some of the problems which is given in Cengel Cimbala or F.M. White book in exercise and example problems. With this, let me conclude this lecture today. Thank you a lot.